Acoustic localization errors estimation : relevant parameters study

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ABSTRACT
The localization performances of a two-step spatio-temporal goniometry are estimated thought the determination of the relevant parameters that introduce localization errors. A statistical error estimation method is validated by handling a simple two-sensor antenna in a 2D localization and applied to the 3D goniometry with a 3D antenna.

INTRODUCTION

This work was motivated by the increasing need for acoustic localization systems. Various localization systems were implemented during a PhD. These include localization of snow avalanches, artillery and supersonic aircraft in the infrasound domain, the localization of helicopters, civilian aircrafts, speakers and auditorium reflections in the audio domain and the localization of chirps in the underwater ultrasound domain.
The "goniometer" is defined as an instrument that measures angles. An "acoustic goniometer" is therefore a system that measures the direction of arrival (DOA) of a sound source, and thus estimates the source direction.
A goniometer is made up of an antenna, composed of several sensors arranged in a particular geometry, and a calculation algorithm.

II. SOURCE LOCALIZATION PROBLEM

The successive implementations of the localization algorithms were designed around a common framework, based on a two-step spatio-temporal process (Figure 1). The temporal step tackles the problem of the Time Delay Estimation along the antenna baselines, whereas the second step introduces the antenna geometry, in order to estimate the Direction of Arrival *per se*.
The time delay estimation is a temporal process as it is based on the generalized cross-correlation techniques. On the other hand, the localization module introduces the antenna geometry and size, the speed of sound and is a spatial process.
The objective of this work is to estimate the performances of the localization module. The qualities and inconveniencies of this two-step method will be highlighted by listing every relevant parameters of the goniometry and by studying their influence on the performances. First, a theoretical approach will handle the simple case of the DOA determination with 2 sensors (one baseline). This two-dimension case will be solved by using both mathematical and statistical methods. Next, the three-dimension case will be handled with the previously validated statistical approach.

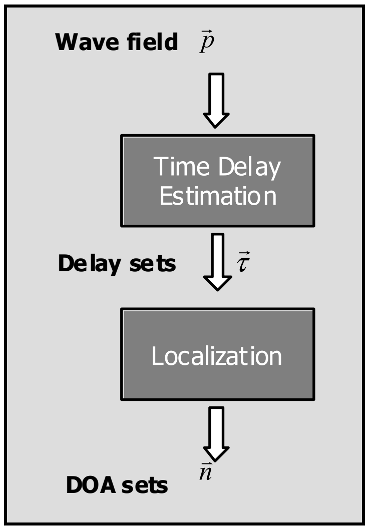


Figure 1. goniometer schematic diagram

III. TWO-SENSOR GONIOMETRY

The simple case of 2D goniometry with a pair of sensors is represented in Figure 2.

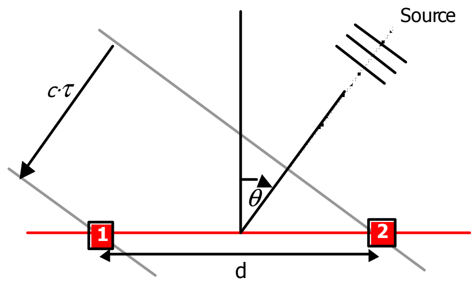


Figure 2. 2D propagation model

In the far-field case, the DOA is completely defined by the angle θ and is obtained by the relation :

$$\theta = \sin^{-1}\left(\frac{c\tau}{d}\right) \quad (1)$$

To observe the robustness of the DOA estimation in the presence of the parameter errors $\{\delta c, \delta l, \delta\tau\}$, (1) was developed into a Taylor Series around the nominal values $\{c_0, d_0\ \tau_0\}$. The resulting nominal DOA is given by

$$\theta_0 = \theta(c_0, d_0, \tau_0) \tag{2}$$

First, each parameter is separately contaminated by errors. In a second approach, all parameters are supposed noisy, which implies the introduction of a multi-variable Taylor development.

If we consider that the array geometry and the speed of sound are precisely known ($\delta l=0$ and $\delta c=0$), the error on the DOA is only due to the imprecision on the Time Delay Estimation (TDE) and can be expressed by

$$\delta\theta = \theta(c_0, d_0, \tau_0 + \delta\tau) - \theta_0 \quad = \sum_{n=1}^{\infty} \left\{ \frac{(\delta\tau)^n}{n!} \cdot \frac{\partial^n \theta}{\partial \tau^n} \right\}_{\tau=\tau_0} \tag{3}$$

The determination of the upper limit of the error on the DOA is very important to evaluate the overall performance. If we admit that the precision of the TDE is better than 5% of the maximum lag (physically expectable $\tau_{max}=d_0/c_0$), then we can calculate the DOA error $\delta\theta$ as a function of the nominal DOA θ_0. The next figure presents the mathematical results described by (3) compared with a statistical approach. Indeed, by simulating noisy TDE and by calculating the final DOA error over thousands of attempts, the statistically estimated errors on the DOA present an upper limit theoretically identical to the mathematical approach. The problem is expressed as:

find $\delta\theta = f(\delta\tau)$
with $d_0=1m$, $\delta l=0$, $c_0=342 ms^{-1}$, $\delta c=0$
for $\theta_0 \in [0,90°]$, $\delta\tau \in [-0.05\ d_0/c_0, 0.05\ d_0/c_0]$

For reasons of symmetry, the region $\theta \in [0,90°]$ is sufficient to describe all the phenomena.

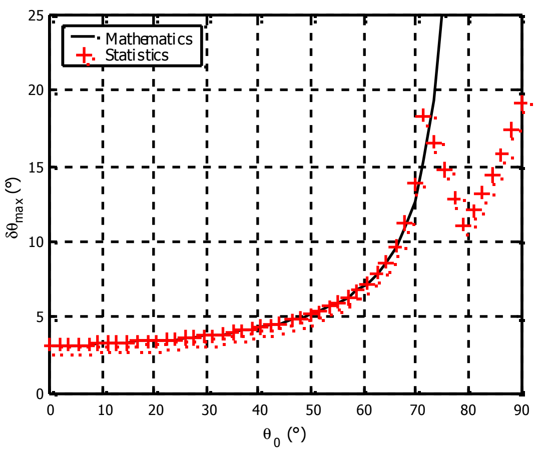


Figure 3. DOA error bound as a function of the DOA angle θ_0 for $\delta l=0$, $\delta c=0$, $|\delta\tau|< 0.05\ \tau_{max}$

The analysis of Figure 3 reveals that the goniometry for sources located in the region around the Broadside direction, performs much better than for sources located in the End-Fire region. In other terms, the DOA sensibility to TDE errors is much greater for sources aligned with the baseline. The DOA performances remain acceptable for a DOA between –50° and 50°, and decrease seriously beyond to reach a maximum value of 20° for θ_0=90°. The saturation of the errors between 70° and 90° observed in the statistical approach, is due to elimination of goniometry resulting in complex DOA (sin(θ)>1). The complex DOA values, corresponding to TDE greater than the maximum physical time delay, are set equal to 90°. The mathematical approach does not take this physical limitation into account. Finally, the slight differences observed around θ_0=70° are due to finite length of the Taylor series expansion used for the mathematical model (Order 4).

A similar development can be carried out in case of inaccuracy either on the speed of sound or on the sensor location (see Figure 4 and Figure 5). The three parameters influence the DOA errors in quite a similar fashion. Yet, an important difference is observed for the Broadside direction: in the particular case of θ_0=0°, errors on the speed of sound and sensor location do not have any influence on the DOA ($\delta\theta$ = 0).

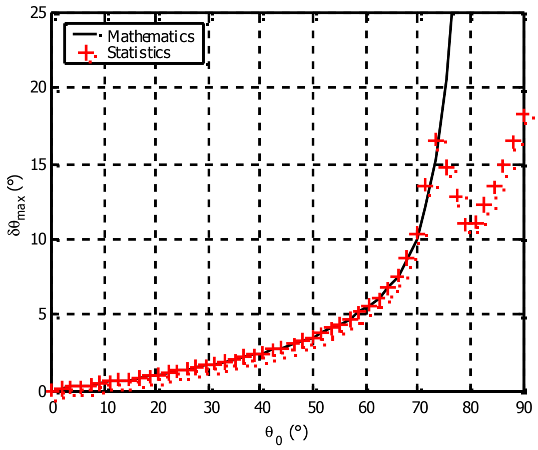


Figure 4. DOA error bound as a function of the DOA for $\delta l=0$, $|\delta c|< 17.1\ ms^{-1}$ $(0.05*c_0)$, $\delta\tau = 0$

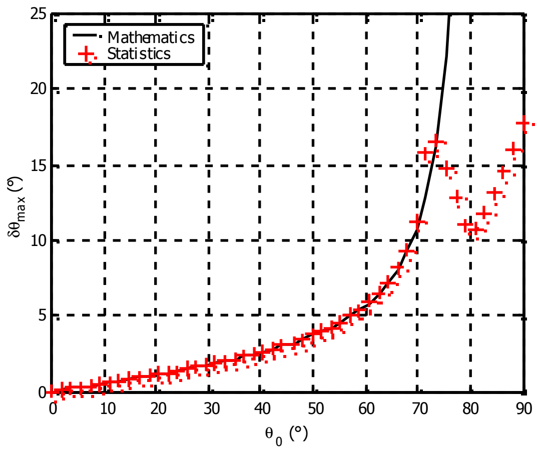


Figure 5. DOA error bound as a function of the DOA for $|\delta l| < 5cm$ $(0.05*d_0)$, $\delta c=0$, $\delta\tau=0$

In the second case, every parameters are contaminated by noise (Figure 6). The mathematical error bound is obtained by calculating the three-variable Taylor series expansion given by

$$\delta\theta = \theta(\tau_0+\delta\tau, c_0+\delta c, d_0+\delta d) - \theta_0$$
$$= \sum_{j=1}^{\infty}\frac{1}{j!}\cdot\left\{\left(\delta\tau\frac{\partial}{\partial\tau}+\delta c\frac{\partial}{\partial c}+\delta d\frac{\partial}{\partial d}\right)^{j}\theta(\tau,c,d)\right\}_{\tau=\tau_0,c=c_0,d=d_0} \quad (4)$$

When all the parameters are noisy, the overall influence on the DOA error is more than the sum of each contribution. As the variables are closely linked (1) cross terms have to be considered.

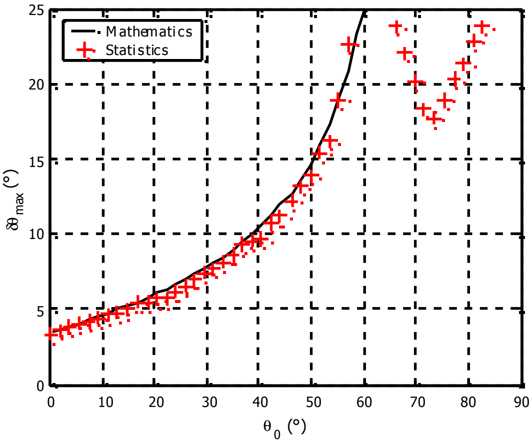


Figure 6. Upper limit of DOA errors as a function of the DOA for $|\delta l|$ < *5cm (0.05*d₀)*, $|\delta c|$< *17.1 ms⁻¹ (0.05*c₀)*, $|\delta\tau|$< *0.15 ms (0.05*d₀ /c₀)*

When performing a two-sensor localization, it appears clearly that the performances fall when the DOA diverges from the Broadside direction. In the case of antenna with more than 2 sensors, the goniometer should be considered as a combination of several sensor pairs with different behaviors.

IV. LOCALIZATION ERRORS

IV.1. Antenna geometry and localization errors

The 3D localization requires at least four sensors located in 3D space. By comparison with the two-sensor case, it is easy to imagine that the localization performances will differ according to the antenna geometry and the DOA. In order to quantify the performance disparities, the singular value decomposition of the relative sensor position matrix is performed. The resulting 3D subspace represents the intrinsic antenna geometry. When the singular values are equal, the antenna presents "revolution symmetry", denoting an isotropic approach. The DOA errors are almost independent of the DOA. The Tetrahedral or the Cube with adequate baseline selection presents this quality. If the condition is not fulfilled, the DOA performances exhibit disparities in relation to the singular values range.

For example, the DOA error bound was calculated for each direction with a four-sensor cube base antenna (Figure 7). The relative bound errors are set to $|\delta l|/d_{max} < 0.05$, $|\delta\tau|/\tau_{max} < 0.05$ and the localization is performed with all six baselines.

The best cases correspond to the DOA presenting the lowest DOA error bound: the direction with azimuth = 180° and elevation = 35.26° is one optimum direction. From the source point of view, the antenna has its maximum of extension (left figure). The worst case can be observed, for example, in the DOA with azimuth = 135° and elevation = 45°, where the antenna shows the smallest extension (see right figure). The performance disparities are, as a first approximation, equal to the ratio between the highest and lowest singular values. In this case, the singular values are [.5 .5 .25] and the ratio=2.

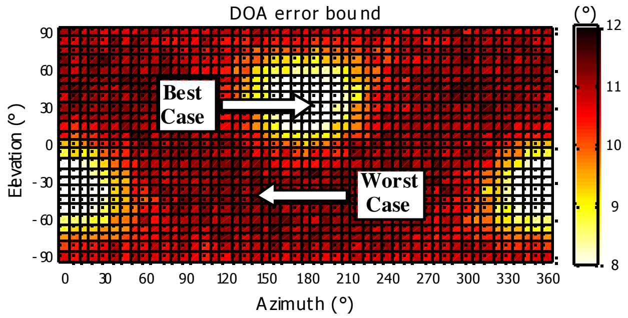
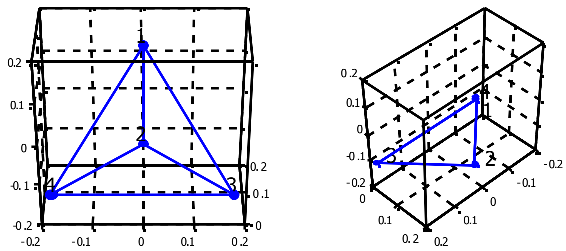


Figure 7. DOA errors bound with $|\delta l|/d_{max} < 0.05$, $|\delta\tau|/\tau_{max} < 0.05$

<u>Source view point</u>



Azimuth = 180°
Elevation = 35.26°
Best Case

Azimuth = 135°
Elevation = 45°
Worst Case

IV.2. Influence of the speed of sound estimation

Despite the fact that the celerity is essential to the knowledge of the sound field, calculations show that the DOA can be totally independent of estimation errors on the speed of sound. This requires the array to be composed of at least 4 microphones located in 3-D space (dimension of the subspace defined by the array equal to three). It can be demonstrated that the errors on the azimuth are independent of the estimation of speed of sound. Similar argumentation can be established for the elevation.

IV.3. Influence of the sensors position estimation

In section III, it was demonstrated that the DOA error varies as a function of the DOA when the relative position of the sensors are contaminated with noise. In order to perform the same analysis for a complex 3D geometry antenna, Monte Carlo simulations were implemented to quantify the influence of the positioning precision on the DOA errors.

The simulations are carried out with the 4 sensor cubic base antenna. Three configurations were investigated (see Figure 8): the first performs the localization by using the 3 baselines of the same length (referred in the legend as "3 independent pairs"), the second and third are using all 6 baselines in the best case DOA and in the worst case DOA defined in section IV.1. The main remark is that the use of all baselines can reduce performance in some DOAs (Figure 9). The goniometry, which is almost DOA independent in the "3 pairs" localization (identical singular values), presents performance disparities when increasing the number of TDE. According to the DOA area to cover, extra precision can be obtained by increasing computation.

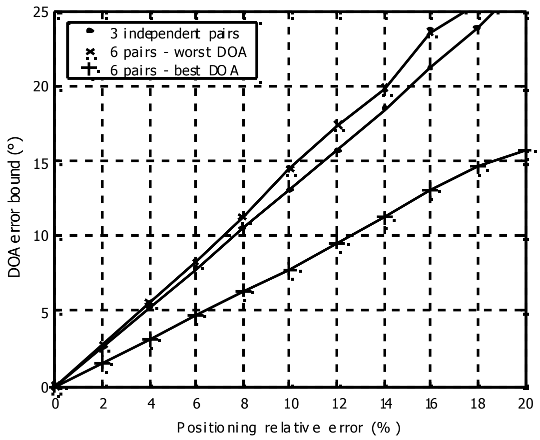


Figure 8. DOA error bound in the presence of sensors position relative error (4 sensors cubic base)

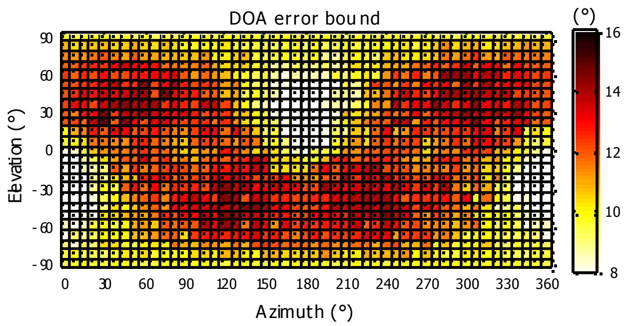


Figure 9. DOA error bound in the presence of sensors position relative error (4 sensors cubic base – 6 pairs) $|\delta l|/d_{max} < 0.1$

IV.4. Influence of the Time Delay Estimation

The influence of TDE errors on the localization is very close to the positioning errors effect, in terms of sensitivity and in terms of performance difference, according to the antenna subspace singular value disparities (Figure 10 et Figure 11).

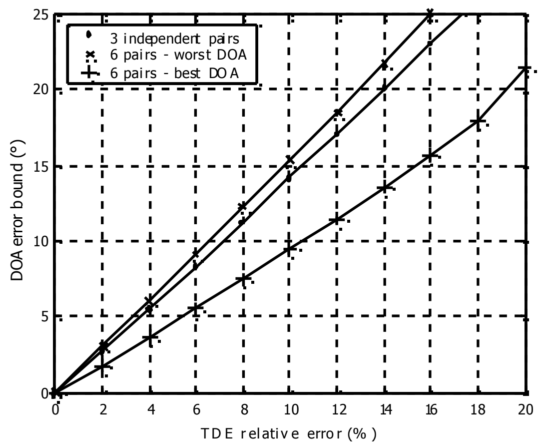


Figure 10. DOA error bound in the presence of TDE relative error (4 sensors cubic base)

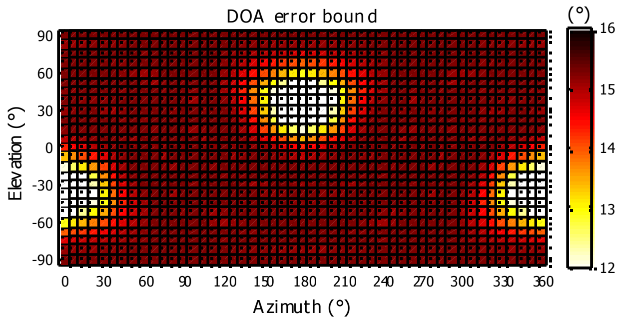


Figure 11. DOA error bound in the presence of TDE relative error (4 sensors cubic base) $|\delta\tau|/\tau_{max}<0.1$

V. CONCLUSION

In order to estimate the potential performances of a goniometry based upon a two-step process (spatio-temporal), the localization errors were studied as a function of the estimation errors on the sensor positions, of the estimation error on the sound speed and of the time delay estimation errors.

The results of the statistical simulations give a rough idea of the performances that can be achieved, and shows that the antenna geometry, the relative position of the source to the antenna, and the baseline selection for the time delay estimation are relevant parameters that modify the localization performances.

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